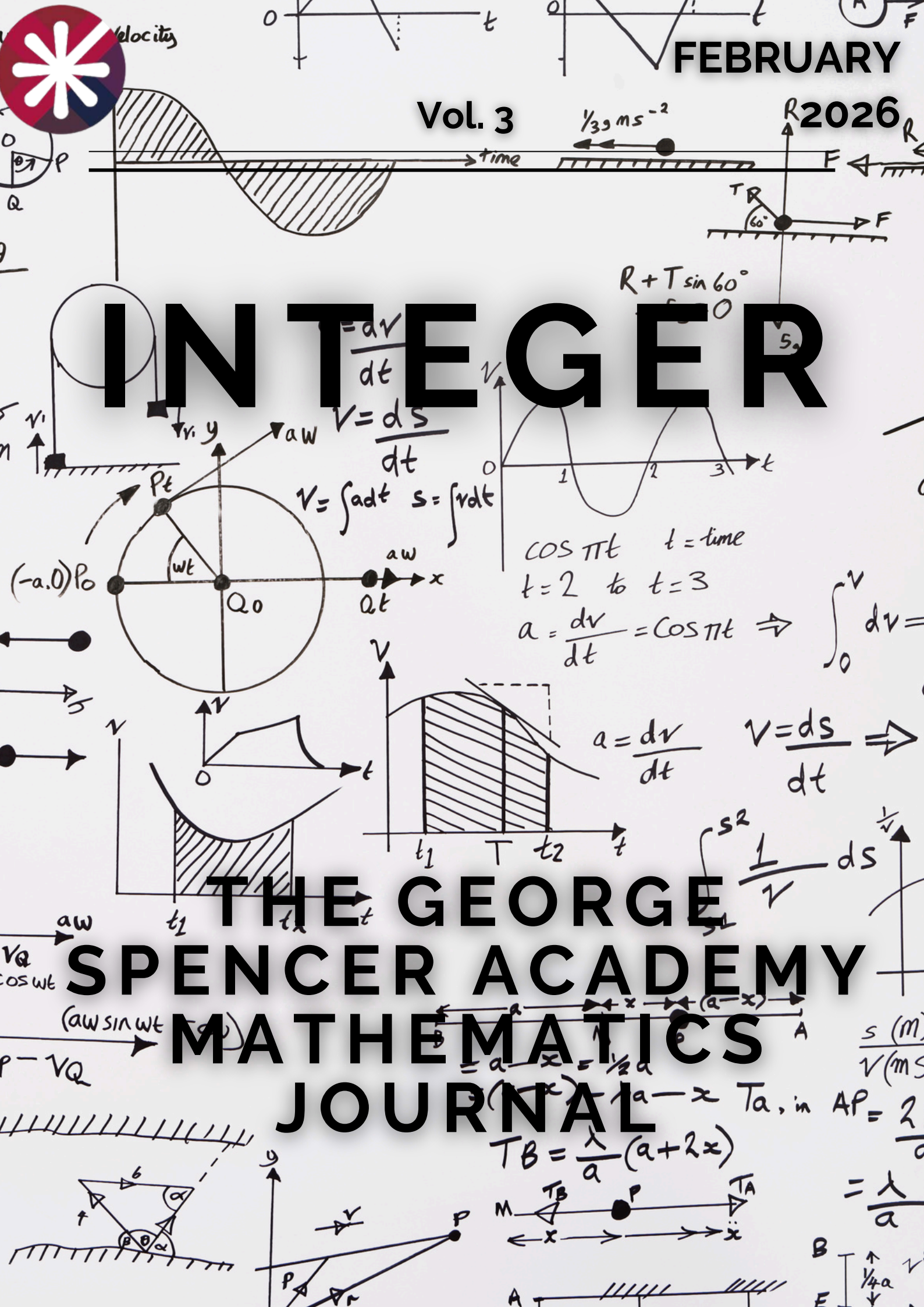


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INTEGER

THE GEORGE SPENCER ACADEMY MATHEMATICS JOURNAL

"Our third edition of Integer showcases the passion our students demonstrate for exploring Mathematics beyond what is taught in the curriculum. If you ever wondered where Maths is used beyond the classroom then this edition has many articles that can answer that question. A variety of topics are covered which can inspire anyone thinking about taking Mathematics at A Level and beyond.."

Miss J. Cullen
Head of Key Stage 5 Mathematics

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WHY THERE IS NO GENERAL QUINTIC FORMULA

Samuel Cooper

Introduction

Most mathematicians know of the quadratic formula, as it is one of the fundamental methods for solving quadratics, along with completing square, factorising, and graphical methods. The quadratic formula will solve any quadratic that is in the form $ax^2 + bx + c = 0$ by the ease of one formula, allowing calculators to solve quadratics for you. This formula is the first general formula for solving polynomials. The following is the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It can only solve quadratics, hence the name of the formula. For higher degree polynomials, there exist more formulae to solve such polynomials. Since there exist a cubic formula and a quartic formula, why is there not a quintic formula?

The Quadratic Formula

The quadratic formula is a universal tool for solving any quadratic equation in the form $ax^2 + bx + c = 0$, providing solutions to such equations. This formula exists as a universal, guaranteed method of solving quadratics, and the formula is derived from this method, completing the square. The following shows the derivation of the quadratic formula using completing the square:

$$\begin{aligned}
ax^2 + bx + c &= 0 \\
a(x^2 + \tfrac{b}{a})^2 + c &= 0 \\
a[(x + \tfrac{b}{2a})^2 - \tfrac{b^2}{4a^2}] + c &= 0 \\
a(x + \tfrac{b}{2a})^2 - \tfrac{b^2}{4a} + c &= 0 \\
a(x + \tfrac{b}{2a})^2 &= \tfrac{b^2}{4a} - c \\
a(x + \tfrac{b}{2a})^2 &= \tfrac{b^2}{4a} - \tfrac{4ac}{4a} \\
a(x + \tfrac{b}{2a})^2 &= \tfrac{b^2}{4a} - \tfrac{4ac}{4a} \\
(x + \tfrac{b}{2a})^2 &= \tfrac{b^2}{4a^2} - \tfrac{4ac}{4a^2} \\
x + \tfrac{b}{2a} &= \pm\sqrt{\tfrac{b^2}{4a^2} - \tfrac{4ac}{4a^2}} \\
x + \tfrac{b}{2a} &= \pm\sqrt{\tfrac{b^2 - 4ac}{4a^2}} \\
x + \tfrac{b}{2a} &= \tfrac{\pm\sqrt{b^2 - 4ac}}{2a} \\
x &= \tfrac{\pm\sqrt{b^2 - 4ac}}{2a} - \tfrac{b}{2a} \\
x &= \tfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

The quadratic formula is known as the first general formula for solving quadratics. The term “general” here means that the formula can be solved by radicals, meaning that the roots can be expressed with arithmetic (addition, subtraction, multiplication, division) and taking nth roots.

The Cubic and Quartic Formulae

The Cubic formula was independently discovered by two Italian mathematicians Sciphone del Ferro and Niccolò Tartaglia, with Gerolamo Cardano publishing it in his 1545 book Ars Magna, leading to it being coined the Cardano-Tartaglia formula. To begin with, Sciphone del Ferro was the first to solve cubic equations, but these were in the form $x^3 + mx = n$, and this is not the general form of cubic equations, being $ax^3 + bx^2 + cx + d = 0$. Del Ferro kept this discovery a secret until near his death,

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telling his student Antonio Maria Fiore.

Niccolò Tartaglia later discovered a broader solution for the general form of cubic equations $(ax^3+bx^2+cx+d=0)$, surpassing Fiore and famously winning a mathematical duel against them.

Gerolamo Cardano persuaded Tartaglia to reveal the formula to him under oath, and he then published the formula in his book Ars Magna, naming it the Cardano-Tartaglia formula, only acknowledging Del Ferro's prior work, which sparked a controversy.

The following is the general cubic equation discovered by Del Ferro and Tartaglia, published by Cardano:

$$
\begin{aligned}
x_1 &= -\frac{b}{3a} \\
&\quad -\frac{1}{3a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d+\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]} \\
&\quad -\frac{1}{3a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d-\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]} \\
x_2 &= -\frac{b}{3a} \\
&\quad +\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d+\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]} \\
&\quad +\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d-\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]} \\
x_3 &= -\frac{b}{3a} \\
&\quad +\frac{1-i\sqrt{3}}{6a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d+\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]} \\
&\quad +\frac{1+i\sqrt{3}}{6a}\sqrt[3]{\tfrac{1}{2}[2b^3-9abc+27a^2d-\sqrt{(2b^3-9abc+27a^2d)^2-4(b^2-3ac)^3}]}
\end{aligned}
$$

The quartic formula has a similar story, whereby Lodovico Ferrari was the one to create this general equation, and his mentor, who happened to be Gerolamo Cardano himself, published the method in the same book as the cubic equation, Ars Magna.

The Quintic Formula

There exist general cubic and quartic formulae, so it is logical to believe that there is a general quintic formula, too, and even higher degrees. However, it has been proven impossible to create one with only basic arithmetic and roots.

Mathematicians did, of course, search for the general quintic formula, and they did manage to create a quintic formula.

This was only for quintic equations in the form $x^5 + ax + b = 0$, known as Bring-Jerrard form, which has a coefficient of 1 for the x^5 term, and is also missing many other terms. These mathematicians managed to find that some form of this equation can be solvable by radicals, but this does not work for all forms of the equation, meaning that there is not a general formula that has been discovered yet.

However, Galois theory shows that while the quadratic, cubic, and quartic equations are solvable by radicals, the symmetric group S5, which is associated with quintic equations, is not a solvable group, meaning that a general formula using radicals does not exist for quintic equations.

Galois theory was introduced by Evariste Galois, and it let him distinguish between the polynomial equations that are solvable by radicals by using the properties of the permutation group of their roots. A permutation group is a group whose elements are permutations of a given set, and the group operation is the composition of permutations. Groups are formed from non-empty sets, and must have an identity element, and each element must have an inverse.

Therefore, an equation is only solvable by radicals if its roots may be shown by a formula involving only integers, nth roots, and the four basic arithmetic operations. This widely generalises and builds upon the older Abel-Ruffini theorem, which was the original theorem that showed that a general quintic formula solved by radicals does not exist. It did acknowledge that some equations are solvable by radicals, but Galois theory will show which equations do.

Conclusion

In conclusion, while it does make sense that there should be a general quintic formula, and general formulae for higher degree polynomials, due to Galois Theory, there can only be some forms of higher degree polynomials that can be solved with radicals.

“WHILE IT DOES MAKE SENSE THAT THERE SHOULD BE A GENERAL QUINTIC FORMULA, AND GENERAL FORMULAE FOR HIGHER DEGREE POLYNOMIALS, DUE TO GALOIS THEORY, THERE CAN ONLY BE SOME FORMS OF HIGHER DEGREE POLYNOMIALS THAT CAN BE SOLVED WITH RADICALS.”

Samuel Cooper

WHY CAN'T WE DIVIDE BY ZERO?

Austin Hornsby

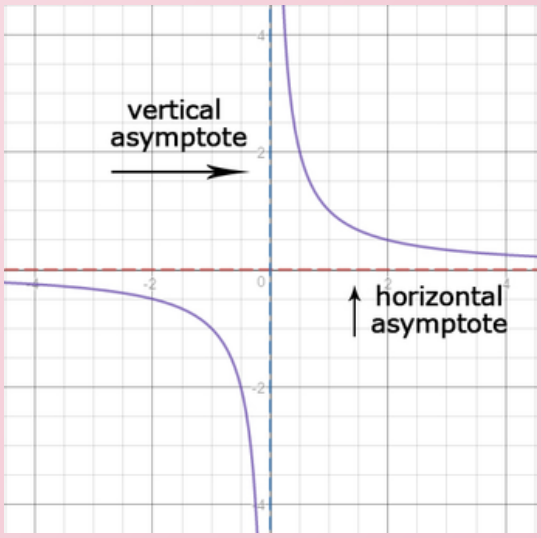
Introduction
One of the most basic yet essential features of mathematics is multiplication and by extension division, yet dividing by zero is still an operation that is not as simple as it might first seem.

Let's try
Now I will go over some methods that try to answer the question.

The Inverse Relationship
To understand why division by zero fails, we can look at its inverse: multiplication. If division is the process of finding a missing factor, then the statement a ÷ b = c implies that b × c = a. When we attempt to divide by zero, we are essentially looking for a number that, when multiplied by zero, returns our original value.

Repeated addition
Breaking it down to its simplest form multiplication is just repeated addition multiplying m by n is adding m to itself n times or multiplying n by itself m times

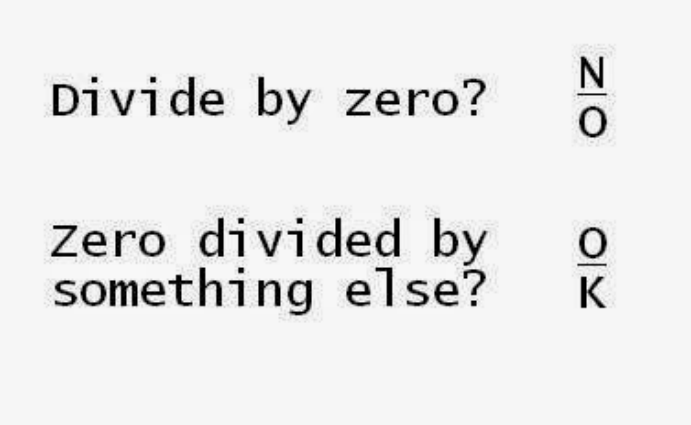
Reciprocals
One of the first things you might think to do when trying to divide by zero is multiplying by the reciprocal. This is since dividing by one number is equivalent to multiplying by the reciprocal; for example, multiplying by two is the same operation as dividing by a half . using this principle of the multiplicative inverse property we can say dividing by zero is the same as multiplying by the reciprocal of zero, which unfortunately for us is unfindable due to the fact that a number and it's reciprocal multiply to one but no number can multiply with zero to make one.



Reciprocal graphs
Since we cannot find a reciprocal for zero we can look towards a reciprocal graph. A reciprocal is when y is equal to the reciprocal of x, giving us these two distinct curves you can see and as it tends to zero we see the y value both increase or decrease exponentially on either side which seems like we are extending towards infinity and negative infinity at the same time so this does not give us a conclusive answer either.

So what actually is the answer?
The answer to what happens when you divide by zero? Is unresolved and practically all attempts to do so defies a law of mathematics when we divide a non zero number by zero we get an indeterminate result this is since if $0 \times x = 5$ there is no possible value for x therefore it is undefined. This is very different to dividing zero by zero as if $0 \times x = 0$ then x has unlimited possible values this means that it is indeterminate. In mathematics, functions are required to have only one output, and since zero divided by zero could be anything, it cannot be defined as a single value.

Conclusion
So in conclusion, while it may appear as though the infamous operation of dividing by zero has a disappointing conclusion, I believe more than anything it shows us that zero is more complicated than you might originally think.



"ZERO IS MORE COMPLICATED THAN YOU MIGHT ORIGINALLY THINK."

Austin Hornsby

THE MONTY HALL PROBLEM: SEEMS SIMPLE, EH?

Oliver Matthews

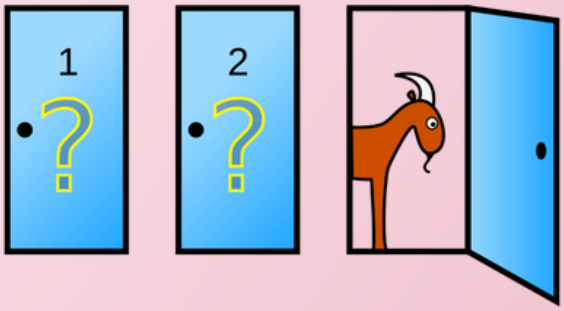
Introduction

The Monty Hall problem is a brain teaser which takes the form of a probability puzzle, this puzzle is named after the host of a 1960s American television show. Not what you expected right? The problem was originally posed in a letter to the American Statistician in 1975. The problem has since garnered much attention and has led to significant debate among mathematicians, statisticians, and the general public. Despite the simplicity of its setup, this puzzle highlights common misunderstandings and misconceptions about conditional probability. The problem is a fairly simple one, I encourage you to think about the problem yourself before reading the rest of my article.

Statement of the Problem and Assumptions

Suppose that you are on a game show, and you are given the choice of three doors: Behind one door is a car: behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? So, what do you think? Stay with the door you chose at the start? Or change to the other?

The classical solution to the problem relies on several key assumptions. The host always knows where the prize is, never opens the contestant's chosen door, always reveals a goat, and always offers the opportunity to switch. The combination of these assumptions ensures that the action of the host provides useful information rather than randomness, which is essential for the probabilistic analysis. It is most effective for you to switch doors.

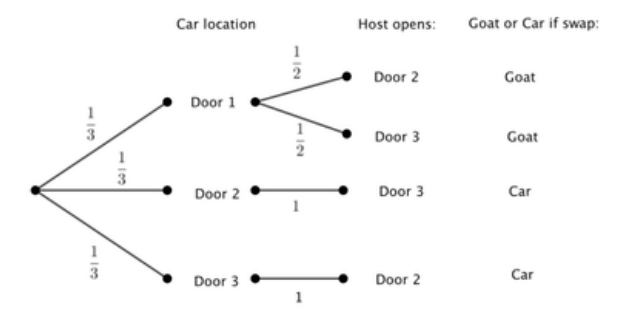


Probabilistic Analysis and Main Argument

For us to analyse this problem we must label our doors. They are labelled as doors 1, 2, and 3, suppose that the contestant initially selects the door 1. Prior to any of the doors being opened, the probability that the car is behind each door is equal, therefore each has a probability of one-third.

If the car is behind door 1, the host must open either door 2 or 3 to reveal a goat. If the contestant switches in this case, the contestant loses. This scenario occurs with probability one-third. However, if the car is behind the door 2, the host is unable to open door 2 based on our assumption and must instead open door 3. This means that switching from 1 to 2 results in a win. In conjunction, if the car is behind door 3, the host is forced to open door 2 and switching results in a win. Each of these scenarios will also occur with a probability one-third.

As such, switching converts the two cases in which the initial choice was incorrect into wins. This then means that the combined probability of winning by switching is two-thirds while the probability of winning by remaining the same is only one-third. This conclusion can also be understood by using conditional probability, after the host opens a door with a goat behind it, the probability that the initial choice was correct remains as one-third, and the remaining unopened door must therefore carry the remaining two-thirds probability.
This reasoning can be formalised using Bayes' theorem, which updates probabilities when new information is revealed; the host's action provides conditional information that shifts probability mass onto the remaining unopened door.



Intuition and Common Misconceptions
A common misconception which needs to be addressed is that many people believe that once one door has been opened, the two doors which remain should then each have a fifty percent chance of hiding the car. The problem herein lies that this intuition treats the host's action as if it were random, which it is certainly not. The host's decision is held by certain constraints such as the knowledge of the location of the prize and the rules of the game itself which means that as the revealed information changes, so does the probability distribution.

Another useful representation of the puzzle is a scenario within a thought experiment which generalises the problem to one hundred doors. After the contestant chooses their door, the host will open ninety-eight doors. This reveals all goats which then leaves only one other closed door. In this case, the initial choice had both probability of one hundredth of being correct which leads us to believe that the remaining closed door must therefore have a probability of ninety-nine hundredths of hiding the prize. This version of the puzzle highlights why the three-door case is not genuinely fifty-fifty and makes the advantage of switching more intuitive.

The Role of Assumptions and Variants of the Problem
Our conclusion that switching is optimal depends critically on the behaviour of the host. If the host does not follow all our assumptions and may sometimes choose a door at random and reveal the prize, or sometimes does not offer the chance to switch, the probability calculations change in their entirety. In such cases the contestant must then consider the host's strategy and apply different conditional probability in accordance with their new assumptions or knowledge.
However, if the host always follows the standard protocol set out earlier by never revealing the prize and always offering the switch; the two thirds probability remains valid. The Monty Hall problem therefore demonstrates the importance of explicitly stating all assumptions when modelling real-world probabilistic systems.

Extensions and Educational Applications
The Monty Hall problem provides an effective classroom example of conditional probability and Bayesian reasoning. One extension of the problem involves

increasing the number of doors to n, in which case switching after the host opens n-2 goat doors will yield a probability of (n-2)/n of winning. Another extension involves running computer-based simulations to reinforce the theoretical result and help students develop intuition about randomness and probability distributions.

Beyond game shows, similar reasoning appears in real-world decision-making problems, such as medical testing, economic forecasting, and artificial intelligence systems, where new information continuously updates estimates of probability.

Conclusion

Under the standard assumption of the host's behaviour, the rational strategy of the Monty Hall problem will always be to switch your door, as this increases the probability of winning from one-third to two-thirds. The problem serves as a demonstration of conditional probability and highlights how additional information alters probability assessments. Beyond its value as entertainment, the Monty Hall problem stands as a valuable pedagogical tool for illustrating probabilistic reasoning, modelling assumptions, and the gap between intuition and mathematical reality.

"THE RATIONAL STRATEGY OF THE MONTY HALL PROBLEM WILL ALWAYS BE TO SWITCH YOUR DOOR, AS THIS INCREASES THE PROBABILITY OF WINNING FROM ONE-THIRD TO TWO-THIRDS."

Oliver Matthews

THE BEAUTY OF FIBONACCI

Enzo Morleo

What is Fibonacci?
The Fibonacci sequence is one of the most well-known sequences in mathematics founded by an Italian mathematician Leonardo of Pisa, known as Fibonacci. The sequence was first discussed in his book "Liber Abaci" published in 1202.
The Fibonacci sequence is a series of numbers where each number is the sum of the two before it. The sequence begins as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

And so on.

It can also be written recursively by

F0=0 , F1=1, Fn=Fn−1+Fn−2F0=0 , F1=1, Fn=Fn−1+Fn−2

Due to recursion, it can be written in programming languages particularly in the functional programming language Haskell, it can be defined as:

```
fib :: Integer -> Integer
fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)
```

Patterns in Fibonacci
The first pattern that can be noticed is the parity pattern. It repeats every 3 terms:
Odd, odd, even
From before:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

This occurs due to rules of mathematics:

- Odd + Odd = Even
- Odd + Even = Odd

The second pattern is the sum of the First n Fibonacci numbers.
This can be written as:

Σk=0nFk=Fn+2 − 1Σk=0nFk=Fn+2 − 1

This means that the sum of all Fibonacci numbers up to FnFn is equal to two terms after FnFn subtract 1.

This allows mathematicians to easily add terms of the Fibonacci sequence instead of adding dozens of numbers.

The final pattern is that each term is approximately 1.6 times the prior term.

The golden ratio
The Greek letter phi **ϕ** is used to represent the irrational number.

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$$

This is known as the golden ratio and is deeply connected to the Fibonacci sequence and explains why each term is roughly 1.6 times the previous one.

Binet's Formula
Binet's Formula is a closed form formula used to find the nth term of the Fibonacci sequence. Derived by the mathematician Jacques Philippe Marie Binet in 1843.

It can be written as:

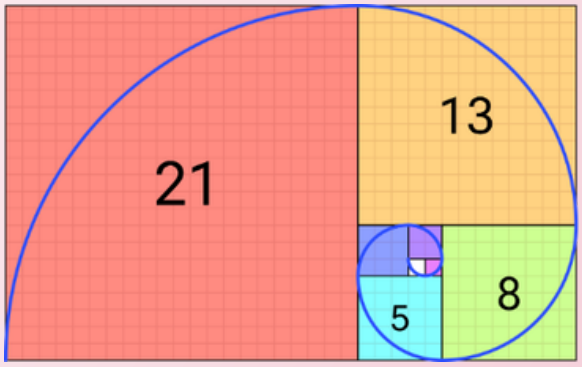
$$F_n = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$$

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The formula can be used to calculate any Fibonacci number directly linking the Fibonacci sequence and the golden ratio.

The formula can be used to calculate any Fibonacci number directly linking the Fibonacci sequence and the golden ratio. Since
$\phi > 1$
the Fibonacci sequence grows exponentially which explains why the numbers increase rapidly as n increases.

The golden spiral expands outward anti-clockwise by a factor of the golden ratio with every quarter turn and if you draw squares like in the image, the side lengths equal to consecutive Fibonacci numbers.



Applications

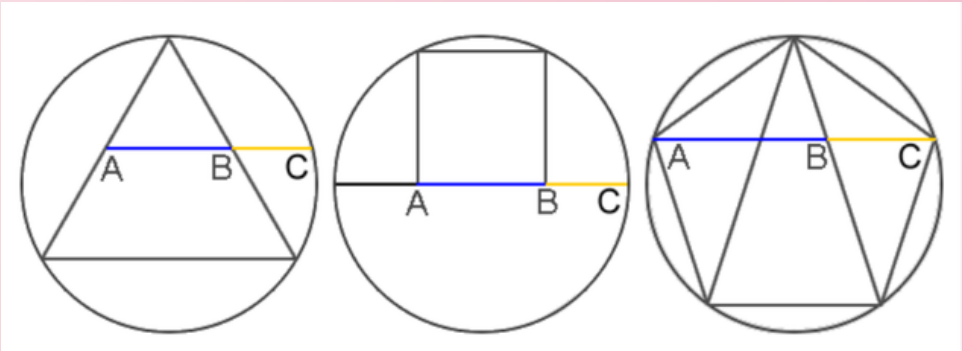
Geometry

The golden ratio is found in geometry, found in equilateral triangle, square and pentagon inscribed in circles. The golden ratio is also found in three-dimensional structures such as dodecahedrons. Even in chemistry, the structure of Buckminsterfullerene also known as "Bucky balls" show proportions related to ϕ.

Nature and Life

Fibonacci numbers and the golden ratio also appear in nature. The number of petals on many flowers, the arrangement of leaves around stems, and the spirals of pinecones and sunflowers frequently follow Fibonacci numbers. Furthermore, the ratio of the important dimensions in animals, and even the spiral structure of human DNA, follow a ratio based on ϕ .





These patterns illustrate that the Fibonacci sequence and the golden ratio are not just abstract concepts; they appear all around us, from the petals of a flower to the coils of DNA.

Conclusion

At first glance the Fibonacci sequence is a basic concept; however, it has deep relations with the irrational number ϕ. From the sum of its terms to the ratios of consecutive numbers, it reveals patterns that link directly to the golden ratio. Across geometry, nature, and even molecules, Fibonacci numbers and ϕ demonstrate that mathematics is not just abstract – it quietly shapes the world around us.

"FIBONACCI NUMBERS AND ϕ DEMONSTRATE THAT MATHEMATICS IS NOT JUST ABSTRACT – IT QUIETLY SHAPES THE WORLD AROUND US."

Enzo Morleo

IS THERE MATHS BEHIND THE RUBIK'S CUBE?

Stevie Curtis



Introduction

The Rubik's cube now exists in many forms, from the 1x1x1 to the 7x7x7 Rubik's cube used in competitions (consisting of 218 pieces!) While the Rubik's cube is incredibly popular, many owners are unaware whether there is mathematics behind the cube, or if solving it is simply due to luck. Through my research, I have found that the Rubik's cube is mostly composed of maths like factorials and group theory, hence why it can be solved with skill rather than pure luck.

Ways of solving the cube

Firstly, maths can be used to calculate the possible ways of solving a 3x3x3 cube. There are 8 corner pieces that can be arranged in 8! ways (as it is 8x7x6x5x4x3x2x1 based on the available corner units). Each corner can be arranged in 3 orientations (as there are 3 sides to a corner), and so 3^8 combinations. Then there are 12 edge pieces arranged in 12! ways, with each one having 2 faces so 2^{12} arrangements. However, in a standard Rubik's cube, only ⅓ of the permutations mean the rotations of the corner units are correct, and only ½ of them have the same edge-flipping orientation as the initial cube, and ½ of these have the correct cube rearrangement parity. Therefore, there are the following combinations:

$$= \left(8! \times 3^8 \times 12! \times 2^{12}\right) \div (3 \times 2 \times 2) = 4.3252 \times 10^{19}$$

$$= 43,252,003,274,489,856,000$$

Group Theory

Group theory is a branch of mathematics which uses algebraic structures - groups - to understand symmetries and transformations between data (the relation between separate groups of data). A group will satisfy four specific properties:

- Closure: For a and b in G, a*b will be in G
- Associativity: For all a, b, c in G, (a*b) *c = a*(b*c)
- Identity: An element e in G - is the identity - and for all a in G, a*e = e*a = a
- Inverse: For each element a in G, there exists b=a-1, and a*b=b*a=e

This can be applied to the Rubik's cube by:

- Closure: Completing 1 move on any two elements (little cubes) results in another element within the set (a new position of little cubes)
- Associativity: The order of operations (moves) is irrelevant to the outcome, if the sequence is maintained (A*B) *C=A* (B*C)
- Identity: The identity is the solved cube, doing nothing will leave the cube as solved
- Inverse: Every move has an "inverse move," so a clockwise turn can be "reversed" by a counter clockwise turn

There are 6 main groups within this theory:

- Abelian (commutative) - If the group operation is commutative - meaning a*b=b*a for all elements a - b in the group
- Non-Abelian (non-commutative) - if a*b! = b*a - for a, b ∈ G
- Cyclic - if it can be generated by a single element. So, every element of the group can be written as "an," where n is an integer and a is the generator.
- Finite - a group is finite if it has a finite (fixed) number of elements
- Infinite - a group is infinite if there are infinite number of elements
- Symmetric - The symmetric group "Sn" of all permutations of n elements

Subgroups are a subset of items from another group, so given a group G, a subset H of G is known as a subgroup of G if H is also a group under *. Also, commutators and conjugation are tools that create permutations. A commutator is [g, h] = g-1h-1gh, showing there is a lack of commutativity between G and H< Conjugation involves applying the permutation of g, then h, then g-1 (all denoted as ghg-1).

God's Number

God's number is the maximum number of moves required to solve a Rubik's Cube from any scramble. Therefore, God's Number is the smallest n where every cube can be solved in n moves or fewer. God's number is estimated using Group Theory. Starting from the solved state:

- after 1 move: there 12 combinations
- after 2 moves: 12 X 11 configurations (assuming you did not undo move 1)
- after 3 moves: 12 X 1^{12} configurations
- after n moves (n >= 2): 12 X 11^{n-1} configurations

Given there are 43.25 quintillion combinations:

$$12 \times 11^{n-1} \geq 43,252,003,274,489,856,000$$

Therefore, as n is an integer, n is at least 19

Algorithms

A common algorithm (based on group theory) is the Layer-By-Layer (LBL) method:

- Layer one: Create a white cross, where each middle edge top square is white, and the side matches the relevant side centre colour. Then place each corner correctly, with the sides colour corresponding to the side centre colour.
- Layer two: the four edge pieces of the middle layer are inserted one by one, using algorithms that pre-exist.
- Layer three: this layer follows a four-step process (step 1: form a cross on the final face, step2: orient the final corners, step 3: swap the final layer edges, step 4: swap the final layer

corners), where other, pre-existing algorithms are used to complete each step. This is repeated until the cube is solved.

Conclusion

To conclude, maths is heavily used in the concept of the Rubik's cube, and group theory is a key concept used to solve more complex cubes. Group theory allows efficient solutions to be formed to a specific cube. This is more efficient than the LBL method, which can be used repeatedly to solve any 3X3X3 cube. Group theory also allows mathematicians to estimate God's number, to find the maximum amount used to solve any one cube. Group theory can even be used to explain how each layer and move will affect every other unit, and all future moves. This is more useful as we progress to more advanced cubes, as the complexity increases (due to more potential combinations), so the number of combinations increases.

"MATHS IS HEAVILY USED IN THE CONCEPT OF THE RUBIK'S CUBE, AND GROUP THEORY IS A KEY CONCEPT USED TO SOLVE MORE COMPLEX CUBES."

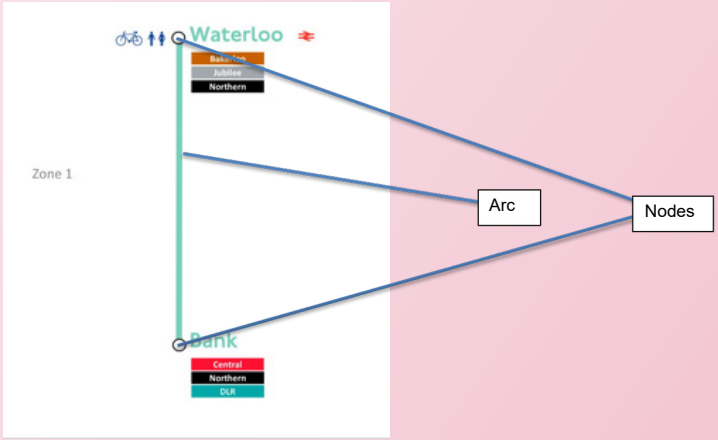
Stevie Curtis

CAN WE UNDERSTAND METRO SYSTEMS THROUGH SIMPLE MATHEMATICS?

Jack Preston

Introduction

168 million. That is the number of people who use underground metro systems in the world per day. A staggering amount. For some it's engrained in their lives, a means of getting to school or work. For others it's a novelty, a way to pack all the sounds and sights of a big city into a day or two. All these people, yet few understand how the myriad of tunnels and trains operate and serve the globe's population. In this article, basic maths is applied to one of the most integral transportation methods to enable the reader to understand how metro systems are engineered to transport millions efficiently every day.



Network and Degree Theory

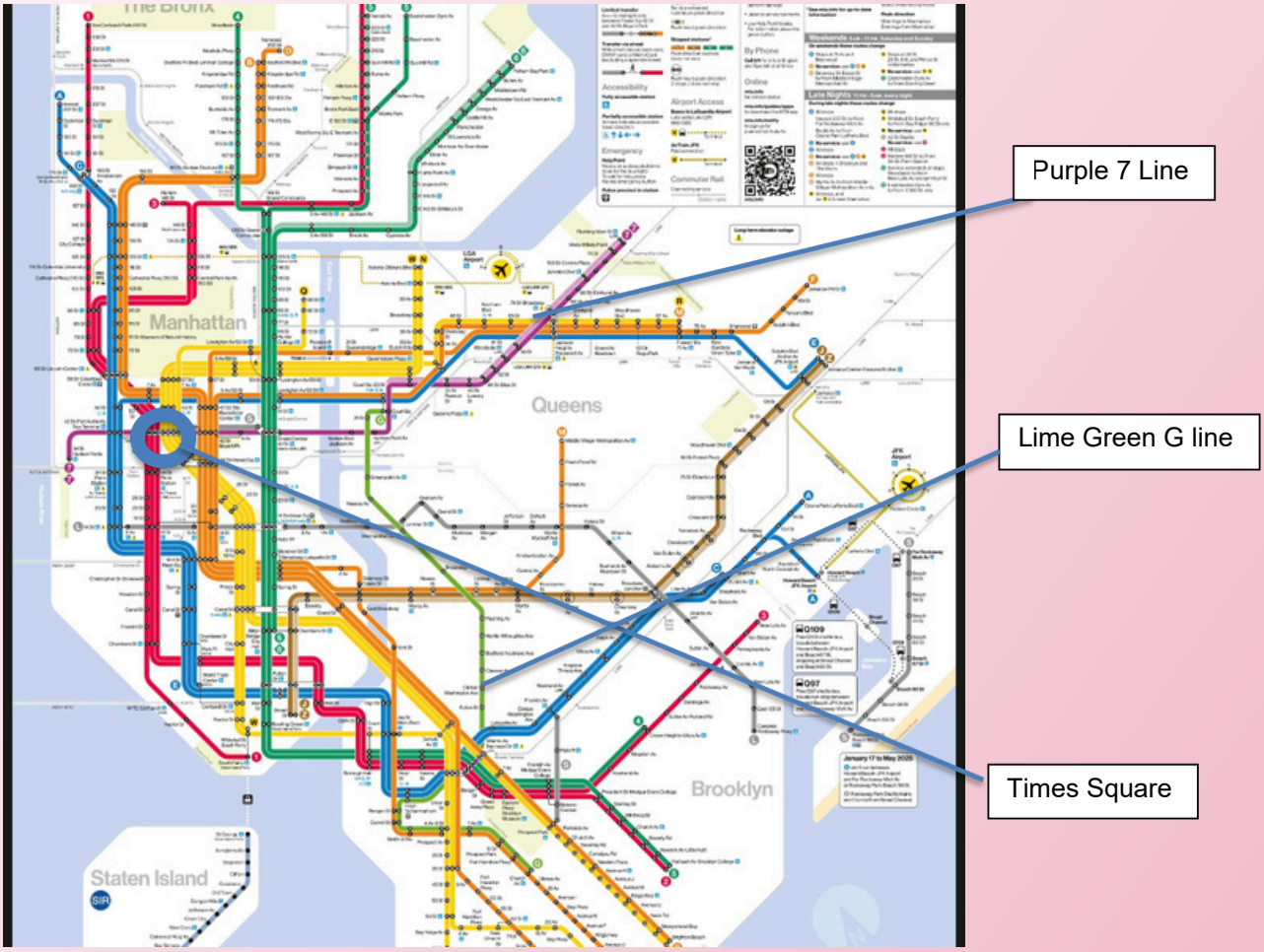
Metro systems can be presented fundamentally as networks - the stations are the nodes, and the trains/tunnels are the arcs. For example, on the Waterloo and City line in London, Waterloo station (node) is connected by 1.5 miles of tunnel and track (arc) to Bank station (node). Using this concept of nodes and arcs, metro planners can make the most vital stations easier to get to by increasing the number of arcs (remember – arcs are tunnels and tracks) – this is explored further below.

Firstly, to assist reader context and to visualise networks, nodes and arcs generally, please see the diagram below: -

Building further on the concept, Metro systems consist of a number of different lines, each serving different areas of their respective city. In New York, the purple 7 line serves Midtown and Queens, whereas the lime green G line runs throughout Brooklyn. Whilst some minor stations are only served by 1 line, planners have multiple lines run through the busiest stations to make them more accessible – akin to how multiple nodes can link to a server. Because of this, each station can be measured by its degree (number of stations it directly connects to) which corresponds to its location and importance. Terminus stations have a degree of 1, insignificant outer stations have a degree of 2, intermediate stations have a degree of 3-4 (2 lines), and hub stations have a degree of 5+ (3 or more lines). Using this theory, a logical prediction is that Times Square should be the most significant and busiest station in New York as it has a

degree of 9, being linked to 9 stations via the A, C, E, N, Q, R, S, W,1,2,3 and 7 lines. The prediction is correct - Times Square is the busiest metro station in New York, and one of the busiest in the world, with an annual passenger count of 57.7 million.

To assist reader context please see the diagram below: -



The idea can be applied to even greater extents, as demonstrated by Chris Solarz and Matt Ferrisi. Using degree theory, they devised a plan to set the world record for the quickest time to visit all 468 NYC metro stations. They identified and wrote down the degree of each station on the metro (subway) map, showing them, which stations allow access to more than 1 line (a degree of 3+). Using this, they devised a route that minimised visits to insignificant stations (degree of 2) as they would travel out to the terminus of the line, then reverse until they reached the first intermediate or hub station (degree 3+ and 5+ respectively), allowing them to switch lines and travel to a separate part of the city before repeating the process again. This greatly reduced their time spent at already visited stations, and furthermore the priority placed on high degree stations made line switching and access to unvisited stations far easier than previously considered. Their ingenious efforts paid off - they set a record of 22

hours and 51 minutes, the first ever time below 24 hours, proving how viewing the system as a network can unlock huge understanding.

Efficiency

When constructing a metro system, the main aim is to provide rapid, convenient transit that gets people to their destination with as little disruption and effort as possible. To measure this, 2 measures are considered, each relating to time and convenience respectively:

Measure 1: Average journey time. To calculate the average journey time, the equation:

$$j = \frac{t}{r}$$

can be used, where j is average journey time, t is total time spent on the metro by all riders, and r is the number of trips taken.

The Metropolitan Transport Authority of New York (MTA) estimates that 615 million hours are spent on the New York metro a year by 1.2 billion riders,

$$\frac{615,000,000}{1,200,000,000} = 0.5125 \text{ hours}$$

which is equivalent to around 31 minutes average journey time – for clarity that means the average journey on the metro takes 31 minutes. Contrast this to metro systems elsewhere – the New York metro is faster than the average commute time of 33 minutes in each of Chicago and LA. However, this measure is somewhat unreliable - for example the majority of journeys in New York are either quick hops around Midtown Manhattan or hour-long commutes from the suburbs. Because of this, there is a second measure which metro planners use to ensure efficient journeys.

Measure 2: Number of stations in journey. When constructing the Washington DC Metrorail, the architects wanted to ensure that the average distance between stations was optimum for rapid travel. To do this, they used Vukan Vuckic's fundamental station-spacing formula to solve an optimisation problem.

In basic terms – Travel Time = Walking Time + Vehicle Time. However, there is a difference between wasted vehicle time (decelerating/stopping) and useful vehicle time – the differential lead to the creation of a new formula (Vuckic's formula).

It states that:

$$(T(walk) = \frac{d}{v}) + (T(vehicle) = \frac{t\,(stopping)}{d})$$

where T = Wasted time, d = average distance between stations, v = walking speed (1.4m/s) and t = time spent not at max speed due to stopping at a station (~60 seconds per stop in the case of the DC metro) (per km), is equivalent to total time wasted.

This combines to create the Vuckic's fundamental station-spacing formula:

$$T = () \frac{d}{v} () \quad \frac{t}{d}$$

To optimise this equation, the first differential must be found:

$$\frac{dT/}{dd} = \left(\frac{1}{v}\right) - \left(\frac{t}{d^2}\right)$$

As the value is a stationary point (minimum), the equation can be set equal to 0, and rearranged to produce a formula for d.

$$\left(\frac{1}{v}\right) - \left(\frac{t}{d^2}\right) = 0$$

$$\left(\frac{1}{v}\right) = \left(\frac{t}{d^2}\right)$$

$$1 = \left(\frac{tv}{d^2}\right)$$

$$d^2 = tv$$

$$d = \sqrt{tv}$$

As t has already been established as 60s and v as 1.4m/s,

$d = \sqrt{60 * 1.4} = 9.2$ *metres*

However, this value is per passenger on 1 train. The architects concurred that around 120 passengers would use each train on average at once, and therefore multiplied the initial value by 120, giving a value of 1,104m or 1.104km on average between each station for the optimal station spacing.

50 years on from its construction, the DC Metrorail is regularly ranked as one of the top 5 rapid transit systems in the USA for organisation and passenger convenience despite being the second busiest, perhaps thanks to Vuckic's work. Many rapid transit systems constructed since the DC Metrorail apply Vuckic's formula to optimise station spacing, including the Singapore MRT and many others.

Conclusion

In conclusion, this article makes it clear that Rapid Transit Systems are constructed and carefully thought through using mathematics, and therefore can be understood through the application of mathematical knowledge and concepts – whether basic (such as via the concept of a network), or more complicated (such as differentiating to optimise station spacing). However, it is important to understand one limitation of maths here - the randomness of humankind. By way of example, Vuckic's formula does not account for the construction of an American football stadium in Washington DC in 1997 which puts huge pressure on a small part of the Washington DC metro throughout winter weekends (football season), nor does degree theory account for the droves of people that fill up Tottenham Court Road station (degree of 2) after the theatre has finished for the night, making it the fourth busiest tube station in London. As with a great deal of things in life, maths may help us to understand metros and devise theoretical solutions and efficiencies BUT it does not properly allow for the chaos and pattern of human nature.

GAME THEORY IN THE PRISONER'S DILEMMA

Bradley Gallacher

Introduction

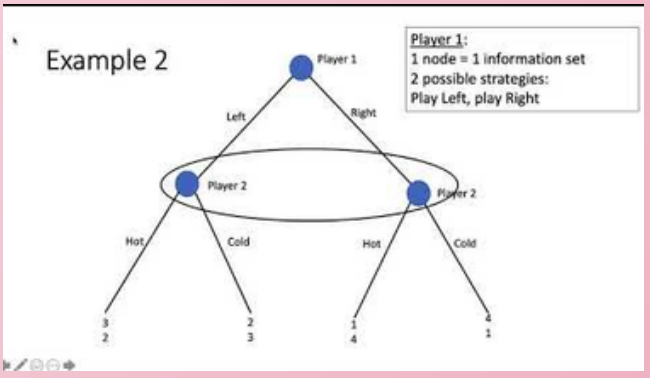
Game theory is something that everyone uses in their day-to-day life even without thinking about it or realising that you are doing it. There isn't an action that you could do with another person that would result in you not partaking in game theory because even trying to actively reject and avoid game theory is contributing to game theory. Game theory is used in a multitude of ways in a variety of subjects like economics and biology and is displayed and calculated using many mathematical methods. In biology it was used by Maynard Smith and Price in 1973 to analyse how a conflict between animals of the same species, also known as 'limited war', can benefit individual animals while also benefitting the species. In economics an example of game theory is a price war when 2 firms could hike their own prices for their own benefit with the risk of both firm's prices dropping or stay at a stable price and hope the other firm doesn't hike their prices.

What is game theory?

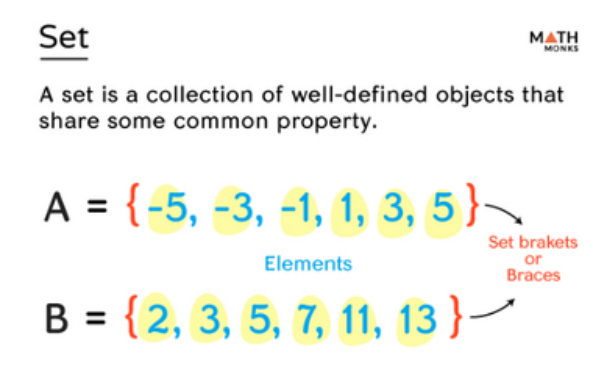
The game aspect comes from relating real life components into 'game terms'. For example naming each person involved in the game as players and potentially having a strategy, rules and there being payoffs involved. Game theory is weighing up the gains and losses from an interaction with another human being. It could be as simple as two people being nice to each other in a conversation which would result in both people having a positive outcome in the interaction to something as complicated and morality testing as the prisoner's dilemma.

Mathematical Implementation

There are a few maths-related aspects behind game theory. For example the 'game terms', as mentioned earlier, the players can be elements of a set, which is a key component which the 'game' is based around. Strategies can be elements in a set, with each strategy being a number which is an option for each player. Payoffs can be functions that map strategy combinations to numbers and matrices can also map and visualise a players strategy and pay offs. An example of setting the players into sets could be shown where there are different choices to be made by different players as seen in the example below.



In this image, Player 1 is an element of the set, which is both players' options and strategies, and at each split path in the flow chart is element of the set which can be represented as a list of numbers in a curly bracket like in the image below.



A matrix can be used to show all of the players their available strategies, as presented by the rows and columns, which is clear and easy to read or understand.

The Prisoner's Dilemma

What is probably the most well known example of game theory is the prisoner's dilemma which is a hypothetical situation where two people who can either cooperate for mutual benefit or betray each other for personal, individual gain. Both people are sitting in separate rooms and they are given the choice, they can either admit that they committed the crime, testify against the other or stay silent. If one testifies against the other then the testifier gets to go free and the other spends 5 years in prison.But If they both stay silent then they will both spend 1 year in prison. However, if both of them testify against each other then they'll both spend 3 years in prison. The obvious best outcome if you are in that situation is that you run free with no prison sentence, however the best mutual outcome is if you and the other person stays silent and you both spend 1 year in prison. So this begs the question, is it worth risking a bigger punishment if it means you could also receive a worse punishment? But even if you do not receive the worst punishment, is it morally right to give the other person a longer prison sentence?

This is what a matrix of the prisoner's dilemma would look like.The top row of letters C and D represent the choices of player 1 and the left column of letters represent the choices of player 2. With C meaning that the player testified and D meaning that the player stayed silent.

	C	*D*
C	(3,3)	(0,5)
D	(5,0)	(1,1)

The top left set of numbers is the result of both players testifying against each other. The top right and bottom left set of numbers are the result of one person testifying against the other and the other staying silent. The bottom right box is the result of both players staying silent.

Nash equilibrium

The Nash equilibrium was named after its inventor, John Nash, and it is considered to be one of the most important concepts of game theory. It is considered important because it relates to its applicability and can be incorporated into a wide range of scenarios. It is a decision making theorem

within game theory that states a player's best chance of achieving their desired outcome and is optimal when considering the other player's decisions. It can often be discussed in conjunction with the player's dominant strategy which states that the chosen strategy of someone will lead to a better result out of all the possible strategies that can be used and does not take into mind the other player involved and how that would negatively impact them, regardless of the strategy that the opponent uses. It can simply be described as "A scenario in game theory in which no player in a non-cooperative game has anything to gain by changing only their strategy."

Conclusion

In conclusion, I believe that in a scenario like the prisoner's dilemma the best option is to follow the Nash equilibrium which would be to do nothing and hope the other player also does nothing which results in the second best individual outcome but the best outcome for both players.

"THE BEST OPTION IS TO FOLLOW THE NASH EQUILIBRIUM WHICH WOULD BE TO DO NOTHING AND HOPE THE OTHER PLAYER ALSO DOES NOTHING."

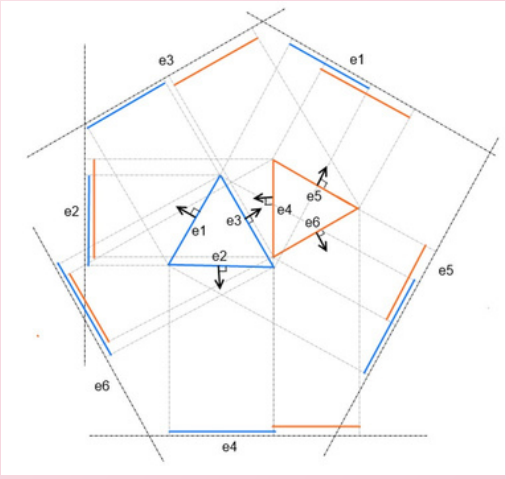
Bradley Gallacher

COLLISIONS IN GAMES

Alex Paton

Introduction
Collision detection is integral to many different applications including physics simulations and video games. Because of this a fast and accurate algorithm is required to make these applications performant and optimised. There are many different algorithms for detecting collisions such as AABB or OBB. Both of these algorithms use a simplified version of Separating Axis Theorem. Separating Axis Theorem states that if two convex objects are disjoint then there exists a line, the separating axis, onto which the two objects projections are disjoint.

The separating axis
The separating axis is a line perpendicular to the hyperplane that separates the two objects. In 2D, to get the separating axis you use the line perpendicular to the normals of the edges of the objects. Then, you project the convex objects onto this separating line using the dot product of a unit vector on the separating axis and a vector to the point of the shape.



From the projected values, you can find the minimum and maximum point of each shape. If one or more of the projections are not colliding then the two objects are not colliding.

Concave Objects
SAT does not work in all cases. One major flaw is that SAT does not work on concave shapes. However, a concave shape can be split into multiple convex shapes. Then you can test collisions on each of these convex shapes, if one or more of the convex shapes are colliding, then the entire concave shape is colliding.

How do you determine if an object is convex or concave? For a convex shape all of the interior angles will be 180 deg or less. However for a concave shape at least one of the interior angles will be a reflex angle.

Minimum Translation Vector
The minimum translation vector (MTV) is the vector with the smallest magnitude that will move the objects so that they are no longer colliding. To get the MTV, you find the projection axis where the objects have the smallest overlap. This smallest overlap is the magnitude of the MTV. To get the direction of the MTV you use the direction of the axis that gave you the smallest overlap.

Circles and curves
For collision detection with circles, it is slightly different. As a curve has infinite

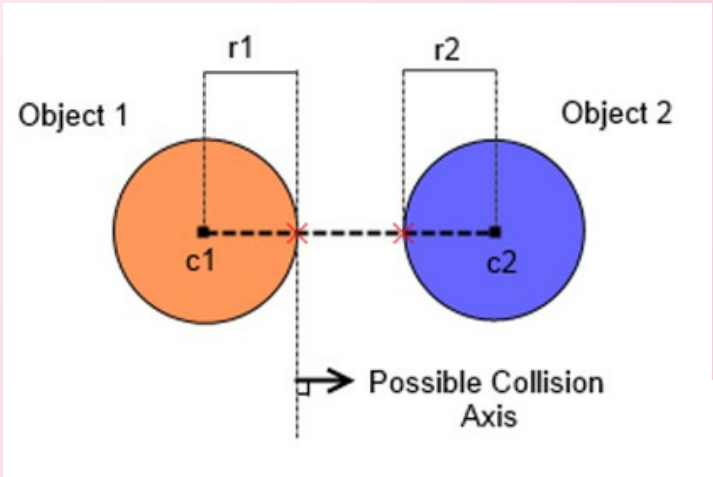
GEORGE SPENCER ACADEMY

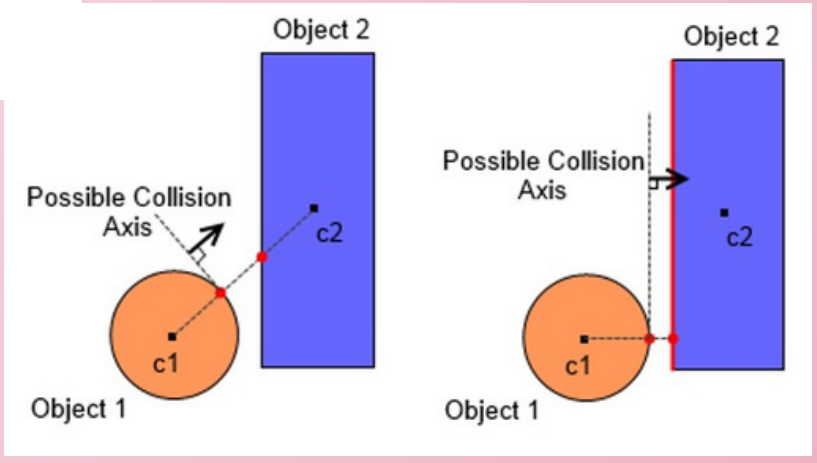
edges and infinite normals, how would you find the separating axes to use? For a sphere to sphere collision check, you have to check whether the distance between the two centres of the circles is less than the combined length of the radii to prove that they are colliding. In this case, the only axis that needs to be checked is direction between those two points.

For a circle and a polygon, it is a bit more difficult First, you would check the normals of the polygon shape, same as with two polygons. However to project the circle you would project the centre of the circle onto the axis and add or subtract the radius of the circle for the range. You can now use this to check if the circle and the polygon are overlapping on the axis. This is also one special case for circle and polygon collision. As a circle is round, it may collide with the corner of the polygon. To check this, you have to identify which vertex of the polygon is closest to the circle's centre. You can create a new axis vector by subtracting the vector of the vertex from the vector for the circles centre and normalising it. You can now use this axis for the final collision check.

How is SAT used?

SAT is used for real time collision calculations for simulations and video games. For video games, SAT is used for the narrow phase of collision detection allowing for checks such as if the player's hit-box has been hit by an enemy. SAT is also used in robotics for collision detection in 2D, SAT allows for robots to avoid hitting obstacles while performing movements.





CAN AI MAKE HUMAN MATHS REDUNDANT IN DRUG DEVELOPMENT?

Olivia Wilson

Introduction

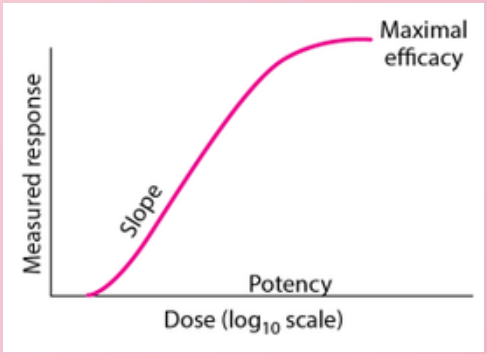
Since the first enzyme was discovered in 1833 (amylase, which helps to digest carbohydrates in the mouth), mathematics has underpinned the foundations of biochemistry, defining kinetics in enzyme theory through to the statistical analysis of data sets in the search for cures of cancer. Maths is especially necessary in drug development, where one inaccurate calculation could mean the difference between life and death. Historically, maths was essential in understanding the complexity of division and digestion within the human body, but with the development of Artificial Intelligence, could AI have beneficial applications in biochemistry, and could it mitigate the need for maths altogether?

Mathematics in Drug Research and Development

Mathematics is central in all fields of biochemistry and biochemical research, elevating drugs from concepts to feasible solutions. Drug development begins with the mathematical analysis of chemical mechanisms determining rates of reaction to explore how the molecules will interact with their biological targets. For example, how the drug paracetamol interacts with pain receptors in the brain. Once the drug has been determined on a molecular level to be viable, mathematical modelling can be used to predict its effectiveness. Complex mathematical and medicinal models are combined to predict the efficacy of the treatment, for example: time conditions such as; how quickly the disease conditions may improve over time, or spatial attributes such as; how quickly a tumour shrinks. Only after the drug has been deemed safe and viable can it be tested through lab and clinical trials, this means that mathematics determines the safety of chemical treatments before being tested on live cells, and then people.

Maths is then integral to ensuring a drug can become publicly available. Statistical data from clinical trials can not only be used to ensure the efficacy of the drug, but also, the data can be used to determine whether side effects are statistically valid. For example, the Chi-square Test can be used to compare the incidences of particular side effects and the Relative Risk to Odds ratio can measure the strength of association between the drug and the side effect.

Maths is also crucial in establishing dosage. Dose response curves can be plotted to depict which dosages have effects that are beneficial enough to be progressed further.



(Farinde, 2021)

A typical dose-response graph (as seen above) includes a logarithmic scale along the x-axis for dosage plotted against the drug response. This enables the potency and efficacy to be compared between drugs, if they were tested under identical conditions. The potency is the x-intercept, and indicates the dosage required for the drug to have an effect. The dose response curve then plateaus, and this signifies the maximum response the drug can have. The dosages between this stationary point and the x-intercept can then be put through clinical testing to confirm a dosage that maximises efficacy and safety. This also helps scientists to understand the characteristics of those who will benefit most from the drug, as dose response curves from different clinical trials can be compared, helping determine when to prescribe it. As well as confirming which dosage sizes are lethal and which are safe.

Can AI be integrated?

AI could be implemented into drug development by processing the statistical data for humans. This would be beneficial because AI can recognise patterns quicker than humans ever could, and find patterns that humans may miss. This means that dosage size will be more accurate and the probabilities of negative side effects from these dosages will be detected much quicker. The consequence in the wider world is that drugs are pushed through clinical trials much more rapidly, meaning they are available to be prescribed sooner, and positively impact and potentially save more lives.

Additionally, AI could use predictive models to explore compounds in the same homologous series (or other specific groups) as existing medicines, to see whether they could be feasible medicines also. Despite AI's role in this, the process of refining and developing the drug would have to be done by humans because, although AI can predict from prior knowledge, it cannot innovate in the same ways humans can.

The use of AI in drug development could be integral in the future, proposing many advantageous qualities. AI is faster and more reliable when completing repeatable tasks such as data analysis. This, combined with AI mitigating human error (within data sets of 10,000 entries, up to 400 can be wrong due to unavoidable human error) makes AI much more efficient for the statistical analysis aspects of drug development. Furthermore, the integration of AI reduces the reliance on human complex mathematics, which can increase accessibility into the field because it removes the barrier of needing strong mathematical abilities (which are commonly considered within society a very binary skillset, of something you either have or do not have, not a skill that you can develop). This increased accessibility can bring a wider range of ideas, increasing innovation.

However, due to drug development being such a sensitive topic, with detrimental repercussions if aspects such as dosages are miscalculated, AI software should be carefully constructed and used in combination with thoughtfully evaluated prompts. Furthermore, the response should be critically analysed and sense checked to ensure that the result is accurate. This means that AI cannot replace human mathematics, only complement it. Although, as AI technology progresses, it may fully mitigate the need

for human mathematics, despite the time and cost limitations meaning it is not able to do so currently.

Conclusion

Overall, mathematics is crucial in every stage of drug development, from an investigative concept, to a prescribable product. Implementing AI into this process can not only increase the speed of drug development but also make the mathematical analysis within it more accurate because AI is more efficient at recognising patterns, spotting anomalies and interpreting statistical data than a human could ever be. This means that the use of AI can speed up clinical trials, pushing drugs into pharmacies quicker, potentially saving lives. Not only can AI accelerate drug testing, it can also explore potential new drugs through predictive models using the homologous series of existing drugs. These predictions can then be tested by scientists, with the capability of becoming successful new medicines. However, despite the benefits of AI, it cannot fully replace human mathematics at this stage of its technological development. AI prompts and responses would have to be carefully constructed and critically reviewed, to ensure the outputs are accurate. Therefore, AI cannot make maths redundant in drug development, only complement the complex mathematics necessary.

"AI CANNOT MAKE MATHS REDUNDANT IN DRUG DEVELOPMENT, ONLY COMPLEMENT THE COMPLEX MATHEMATICS NECESSARY."

Olivia Wilson

WHY ARE COMPLEX NUMBERS USEFUL?

Mia Hinson

Introduction
Complex numbers are numbers that contain a real and imaginary part, written as a + bi, where i is the imaginary unit, the square root of -1. When students learn about complex numbers and hear their teacher say that they are 'not real', they then begin to question how they could ever be used in useful situations as they are just 'made up'. However, complex numbers have many different uses applicable to real life, such as being fundamental for wireless communication, computer graphics, electricity, electromagnetism and mechanical engineering.

Wireless Communication
Wi-Fi is a wave, described by its amplitude (the maximum displacement) and phase (position in its cycle). Complex numbers perfectly represent this as the real part can be used for amplitude and the imaginary part for the phase. This simplifies wave addition and subtraction, which is used in many ways for Wi-Fi.

Techniques like QAM (Quadrature Amplitude Modulation) uses complex numbers to encode multiple bits of data onto a single element, which allows for very fast data transfer. Essentially, complex numbers allow Wi-Fi devices to interpret radio waves efficiently and quickly, which is why modern internet works the way it does today.

Computer Graphics
In 2D graphics, a pixel's coordinate (x ,y) can be represented as a complex number $x + iy$.

This can be used for rotation as when you multiply a complex number by i, it rotates its point 90 degrees and then multiplying by another complex number adds angles. This in turn simplifies rotations massively, meaning that complex numbers increase efficiency in one of the most essential parts of computer graphics, as rotations are used to create animations, position cameras and align images.

In summary, complex numbers are useful in computer graphics as it makes rotation (an essential function in this field) much simpler.

Electricity
Using Euler's formula, you can represent current and voltage as one complex number where the real part represents the voltage and the imaginary part represents the current combining the two. This massively improves mathematical calculations of power networks a lot easier as it makes it simply algebraic, removing the need for trigonometric equations.

If j is the imaginary unit and x a real number, the exponential function says: $ejx=\cos x + j\sin x$

They can also be used to simplify Ohm's Law for AC circuits, with V = IZ given that the values are complex numbers. Ohm's

Law for AC circuits is used in calculations for series/parallel circuits and voltage dividers, otherwise scientists would have to solve differential equations, rather than just algebra.

Overall, complex numbers are useful in electricity to assist in what otherwise would be very difficult calculations, and it is crucial to be making calculations as simple as possible.

Electromagnetism

In electromagnetism, the main elements are the magnetic field and electric field. These can be represented as a complex number where the real part is the electric field, and the imaginary part is the magnetic field.

Phasors in electromagnetism are rotating vectors having the length equal to the peak value of oscillations, and the angular speed equal to the angular frequency of the oscillations, which can be represented using complex numbers as $F=Aej\theta$

Impedance is the total opposition to current and is represented by a complex number with the parts being resistance and reactance, where reactance is different from resistance as it stores and returns energy whereas resistance dissipates it as heat. Impedance can only be calculated as a complex number, and impedance is crucial for increasing efficiency in components.

Essentially, complex numbers are useful in electromagnetism again for simplifying difficult calculations, which is very helpful when trying to make components efficient.

Mechanical Engineering

Complex numbers are used to analyse dynamic systems such as vibrations, control systems and fluid flows.

In vibration analysis, they help model vibrations by using complex eigenvalues (scalar values that represent the amount of a nonzero vector) to determine natural frequencies and damping characteristics of structures without having to use differential equations, via the use of Euler's formula mentioned previously.

In control systems, they are used to analyse system stability and find out how frequencies affect them.

In fluid dynamics, they are used in potential flow analysis for 2D flows to show stream functions and potential velocity.

Overall, complex numbers are useful in mechanical engineering to simulate and analyse many real-world systems.

Conclusion

In conclusion, people do think that complex numbers are pointless, as when being taught they are told they are 'not real', but they are used for many real purposes, many of which increase the efficiency and convenience of the modern world, such as increased Wi-Fi speeds and engineering simulations.

DOES STATISTICS LEAD TO SUCCESS IN SPORT?

Oscar McNarry

Introduction
This report considers the effect of statistics in sport and how teams use them to gain an advantage over the competition. Statistics are used in sport to help produce strategies through data-driven insights, enhancing player performance and optimizing recruitment. All of this leads to a team gaining a competitive edge off the field leading to results on the field. However sport is inherently unpredictable such as when the US team beat the soviet union at ice hockey during the 1980 Olympics with betting odds reaching 1000-1 and when Leicester football club won the premier league in the 2015-16 season with betting odds at the start of the season being 5000-1. This shows that odds and predictions can be beaten by on field performance.

How statistics won the champions league:
Pre 2016-2017 season Liverpool were in a huge drought not winning a premier league title since 1990, despite being one of England's giants. When Michael Edwards became sporting director he immediately made changes moving from a committee based approach to a statistics based approach focusing on player age rather than a performance based approach. This is shown by how in the 2014 summer transfer windows Liverpool brought in players such as Adam Lallana (26) and Rickie Lambert (32). These players are either in or past their prime However once Michael Edwards became sporting director he started buying players with an average age of 23-24 (just before their prime) such as Sadio Mané (24) and Loris Karius (23). This showed immediate effects as when Mané was 22-24 years old he scored 21 goals in 67 games (0.31 goals per game) whereas when he joined Liverpool he jumped up to 90 goals in 196 games (0.45 goals per game) another important signing (momamed salah) jumped from 0.48 goals per game when he was 24 years old which jumped to 0.62 goals per game by the end of his prime. This shows how effective using age as a statistic can improve the quality of signings. This approach led Liverpool to have players during their prime leading them to win the champions league in 2018-19 with Mané having a pivotable role.

How maths created a dynasty;
In the 2012-13 season, basketball team the Golden State Warriors only won 47 games during their 82 game season shooting only 1632 3 pointers. This all improved when the Warriors brought in a new coach called Steve Kerr who came in with a maths based approach which he used to 'solve' the game. He did this by using the principle of percentages knowing that a person who has a 2 point percentage of 50% scores the same amount of points as a 33% 3 point shooter ($50 \times 2 = 100$ and $33 \times 3 = 99$). This turned his team from the 18th ranked team in 3 point attempts to the 4th ranked team. This led to Warriors improving their record to 67 wins shooting 2217 3 pointers which means they shot 585 more shots, an

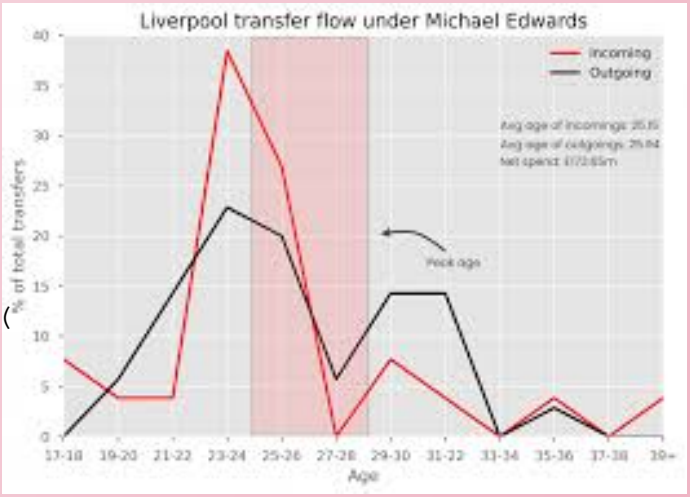
increase of 7 more 3 pointers per game. This turned the team that went out of the playoffs in the first round in 2014 to win the very next year (Steve Kerr's first year). This all culminated in the Warriors setting the single season win record by going 73 wins and only 9 losses creating a true dynasty over basketball winning 3 NBA championships in 4 years only being defeated by the Cleveland Cavaliers lead by Lebron James. This shows how Kerr used statistics to take over the NBA and start the 3 point revolution which spread throughout the league with the total 3 point attempts going from 43000 in 2013 to 58480 in 2017.

How Denmark defeated the odds:

In 1990 Denmark was ranked 21st out of all teams when the euros qualifiers started. This meant that Denmark was not expected to qualify for the euros. Which they didn't. They finished 2nd in their group behind Yugoslavia. However, due to the Balkan war, Yugoslavia were banned from competing leaving Denmark to replace them. Denmark were seen as a filler team who would lose every game this lead to their odds for winning to be 20 to 1. And after a draw to England and a loss to hosts Denmark their odds dropped to 150 to 1. This makes it mathematically impossible for Denmark to go far in the Euros, never mind win. However performances like Henrik Larsen who scored 3 goals (0.6 goals per game) being a huge increase from at club level where he only scored 0.17 goals per game. This show how players can exceed mathematical limits and out perform the statistics and how that's what ultimately decides the game.

Conclusion

In conclusion, there is a huge impact on sport from statistics with them helping to win trophies and create dynasties. Without the use of statistics in sport we wouldn't have the high level of competition in the big leagues, however by teams using statistics the sport becomes inherently less exciting as the way teams play and the results become more predictable because teams are playing as close to perfect as possible. On the other hand this also makes the underdog stories such as Denmark that much more unexpected. So the question is would you rather watch a boring predictable sport with an underdog winning big occasionally or a more unpredictable experience even if it means the underdog rarely wins big as they cannot use statistics to gain an edge over the more skilled opposition? No matter the answer, data and statistics is a growing part of sport as a whole with both companies such as UEFA or the NBA commissioner have started using data to try and make the sport more entertaining and the clubs themself gaining an edge over competition through data-driven insights and the use of data to enhance player performance.



(FC, 2022)

ARE CIRCLES POINTLESS?

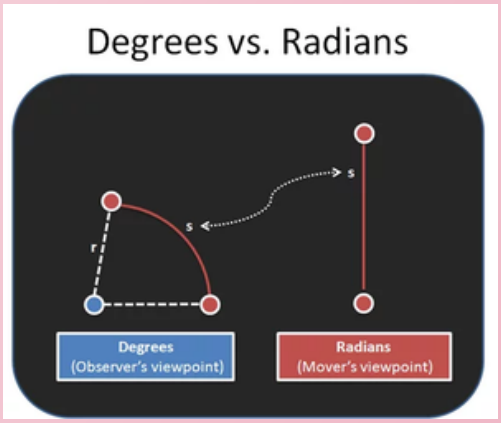
Finn Sanders

Introduction
Circles are everywhere, you learn about them in school, see them in nature and we see them in designs everywhere. Almost everywhere you look you'll find a circle (especially because we live on a huge 3D circle). But why? The answer is because nature loves to be efficient, and circles are very good at that. They enclose the maximum area for the minimum perimeter, a very useful thing when you're trying to be as efficient as possible.

But that's not all circles are good for. Circles are responsible for arguably one of the most important numbers in our universe, 3.1415926535. Also known as Pi.

But where did Pi come from?
It should come as no surprise that Pi has been around for a long, long time. Long enough that we're closer to the year 5000 than the first estimations of Pi by the ancient Babylonians around 4000 years ago. Granted they were fairly far off, ending up with a value of Pi = 3.125, but still an estimation of Pi nonetheless. The Egyptians calculated Pi to be equal to 3.1605, at a similar time to the Babylonians. But the first accurate estimation of Pi was done by Archimedes of Syracuse in 287-212 BC, who determined that Pi is between 3 and 1/7 and 3 and 10/71 , roughly equal to 3.14. He did this using 2 polygons, one that just fits inside the circle and one outside just bigger than the circle. Then by calculating the area of both of these polygons, Archimedes of Syracuse was able to find a rough approximation of Pi.

So where does Pi show up and why do we care?
The first thing that comes to mind is probably angles. More specifically, radians. Radians are a way of measuring angles separate from degrees (which are roughly based on the amount of days we have in a year), where a full rotation is 2 Pi, instead of 360 degrees. This seemingly random number actually comes from the amount of radiuses it takes to cover half of the perimeter. Which makes sense as we know that Pi times the diameter gives the circumference, so it shouldn't be surprising that the radius (half the diameter) times Pi gives half the circumference. So what actually is the difference between them other than the number? In a simple answer, it's how they measure angles.
Imagine watching someone run in a circle around you, degrees measure how far you turn your head to look at them, while Radians measure how far the person moved.



At University, Pi becomes a little stranger. Even with no circle in sight, Pi still appears in an infinite series discovered by Leibniz.

"PI HAS BEEN AROUND FOR A LONG, LONG TIME. LONG ENOUGH THAT WE'RE CLOSER TO THE YEAR 5000 THAN THE FIRST ESTIMATIONS OF PI."

Finn Sanders

It goes like this;

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\cdots$$

and so on with all odd numbers forever. This sequence equals n/4 . An extremely unexpected place for Pi to show up. Furthermore, Euler solved the Basel problem (an infinite series that involves adding up the reciprocals of all square numbers) ending up with a solution of n^2 /6.

Pi is no stranger in the world of physics either, being used in Stefan-Boltzmann Constant which states that the power emitted per unit area by a completely black object heated up to a temperature T is proportional to the fourth power of the temperature.

Conclusion

Clearly, Pi is extremely important, not just when finding the area of circles in class, but also in the universe as a whole. It shows up everywhere, in complex series that seem completely unrelated to anything like circles, all the way to the furthest reaches of space when calculating the expansion of light. So, unfortunately, no matter how much you might hate circles. There's no escaping Pi.

CHICXULUB: THE PERFECT DINOSAUR KILLER

Jerry Weng

1 Introduction

Asteroids hit Earth more often than most people realise. Over billions of years, our planet has been struck by objects ranging from tiny grains of dust to kilometre-wide rocks, many of which have left craters still visible today [3].

But 66 million years ago, one strike was different. An asteroid smashed into what is now the Yucatán Peninsula in Mexico, and within a short time about three-quarters of all species on Earth — including the dinosaurs — were gone [1, 19].

Surprisingly, Chicxulub was not the largest impact Earth has experienced. Other asteroids have created bigger craters without wiping out so much life [11]. Size alone cannot explain what happened.

2 Impact Mechanics

2.1 Projectile Characteristics

Chicxulub was about 10–12 km in diameter with density of ≈ 3000 kg m−3, giving it a mass of ~ 2 × 1015 kg [10, 2]. ≈20kms−1 [8].

Its impact velocity was using the kinetic energy formula: $E_k = \frac{1}{2}mv^2 \approx 3 \times 0 1 J^3$

That's about 75 teratonnes of TNT [10]. For comparison, the total explosive yield of every nuclear weapon ever built is estimated at 10 gigatonnes—less than 0.02% of Chicxulub's energy!

2.2 Trajectory and Angle

The asteroid hit at an angle of 45–60° to the horizontal [10]. Simulations show that impacts in this specific range are the most efficient at transferring energy into the target rocks. This angle maximises surface vaporisation and ejects the largest possible volume of debris and gas into the upper atmosphere, whereas very shallow or vertical impacts tend to either absorb or deflect the energy less destructively [15].

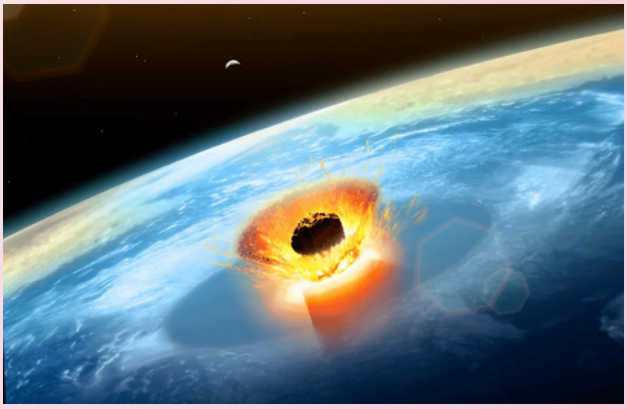


Figure 1: Artist's impression of the Chicxulub asteroid impact. [4]

3 Crater Formation

Crater formation began with excavation, as the asteroid's kinetic energy generated a shockwave that rapidly dug a deep cavity in the crust. This cavity reached about 100 km in diameter and 28 km in depth [13, 14], with rocks briefly behaving like fluid. Subsequent structural collapse caused inward wall failure and central rebound, bringing deep basement rocks to the surface and forming a peak ring. The crater ultimately widened to a final diameter of ~180 km [14].

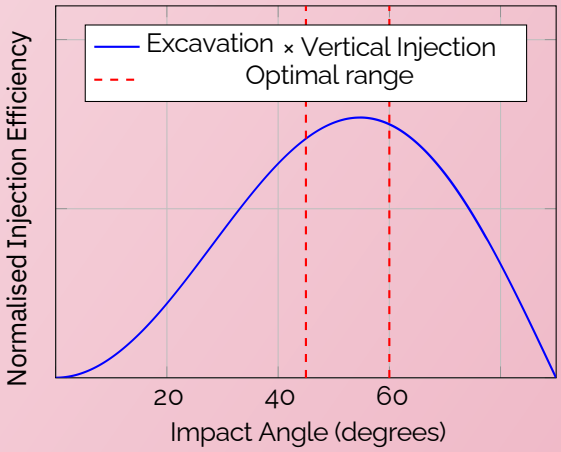


Figure 2: The two combined effects: how much material is excavated (sinθ) and how much momentum is directed vertically into the atmosphere (another sin θ). Their product peaks between 45° and 60°.

GEORGE SPENCER ACADEMY

4 Geology

4.1 Target Rock Composition

The impact site was located on the Yucatán Peninsula, where the crust is made of carbonate and sulfate layers [16]. When these materials were vaporised, they released large amounts of substances, including:

• **Sulfate aerosols:** 67 ±39 Gt of sulfur [9], which reflect sunlight and cool the surface

• **Black carbon:** 7.5 ×1014–2.5×1015 g [21], which absorbs sunlight and heats the upper atmosphere Shock pressures of 25–40 GPa uplifted the underlying crystalline basement rocks during crater formation. This specific mix of rocks made the atmospheric effects much worse than if the asteroid had hit solid granite or deep ocean instead [2].

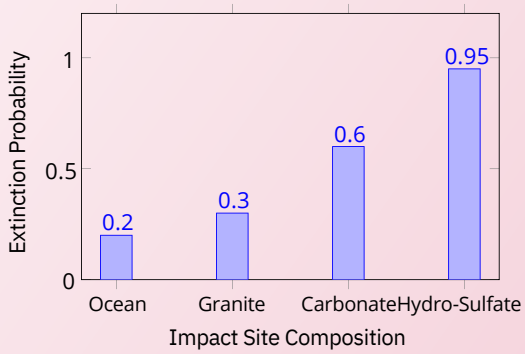


Figure 3: Extinction likelihood from impact geology.

5 Environmental Consequences

Shock waves of 10–100 GPa travelled through rock and ocean water, killing organisms far from the impact site [13]. Tsunami models suggest waves moving at ~200 m s−1 [12]. Atmospheric dust, sulfates, and soot reduced sunlight to ~ 20% of normal levels, which caused rapid global cooling of 15–25°C [20, 9].

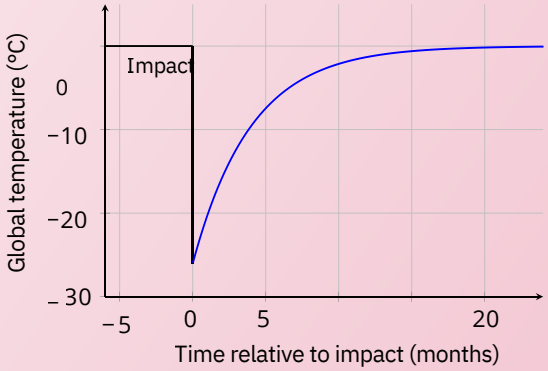


Figure 4: Global temperature anomaly, showing abrupt cooling and gradual recovery.

5.1 Impact Winter Mathematics

The global cooling that followed the Chicxulub impact is can be modelled by an exponential decay function. The temperature anomaly is approximated by

$$\Delta T(t) = -26 \exp\left(-\frac{t}{4}\right)$$

where ΔT is the temperature drop in ◦ C and t is the time in months after the impact. The value 4 represents the time constant τ.

At t = 0 months the drop reaches 26 ◦ C. After 4 months the anomaly has fallen to ≈ −9.6 ◦ C, and after 12 months it is only about −1.3 ◦ C. Exponential models of this kind quantify how long the impact winter persisted and why the prolonged darkness and cold caused ecosystems to collapse so rapidly.

6 Comparison with Other Events

Other large impacts created even bigger craters but did not trigger mass extinctions [11]. For example, the Vredefort impact created a structure ~300 km wide, and the Sudbury basin is ~250 km wide. Despite their immense size, they struck less chemically reactive areas. The Toba super eruption, around 74,000 years ago, released large amounts of SO2 and caused cooling, but still did not result in global extinction [17].

Table 1: Comparison of major impact events

Event	Age(Ga)	Diameter(km)	Extinction
Chicxulub	0.066	180	Yes
Vredefort	2.02	300	No
Sudbury	1.85	250	No
Toba	0.000074	—	No

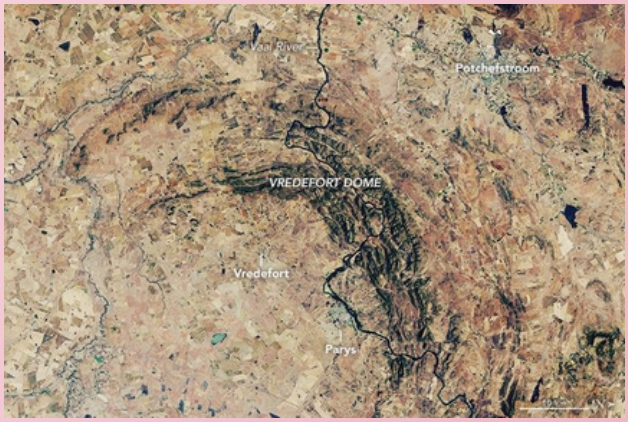


Figure 5: Satellite image of the Vredefort impact structure. Theoriginalcraterrimisdifficulttosee duetoerosionover ~ 2 Ga. (2 billion years) [7]

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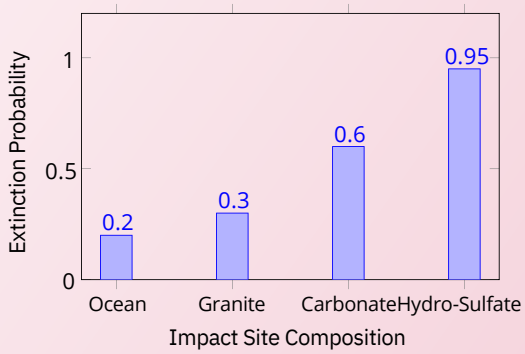


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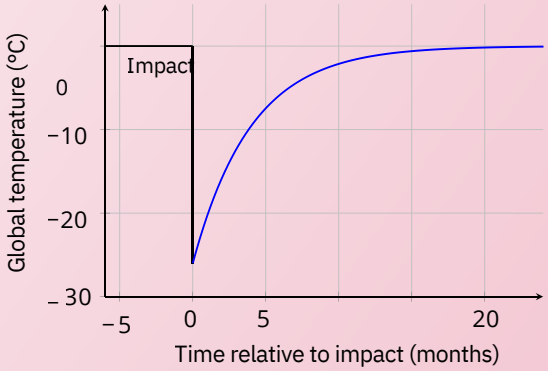


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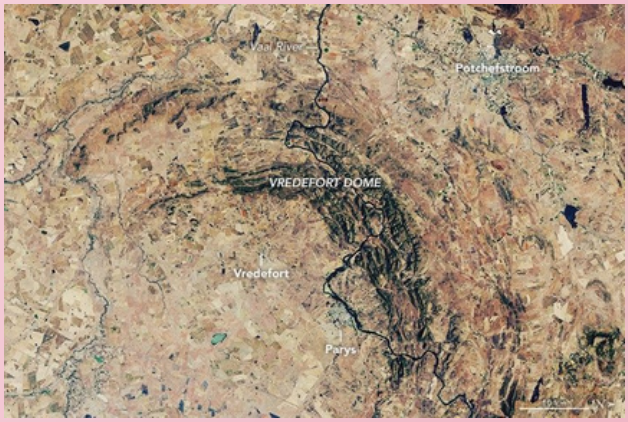


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7 Probability of Catastrophe

Whether an impact causes a mass extinction depends on many separate factors working together at the same time [18, 2]. In the probability calculations below, we use **conditional probability.** The notation *P(A|B)* means "the probability of event A happening, given that event B has already occurred".

$$P(\text{extinction} \mid \text{Chicxulub}) = P(\text{location}) \times P(\text{composition} \mid \text{location}) \times P(\text{angle} \mid \text{site}) \times P(\text{biotic vulnerability})$$

Estimated values are:

$$P(\text{hydrocarbon-rich site}) \sim 0.01\text{–}0.05$$
$$P(\text{composition}|\text{site}) \sim 0.9$$
$$P(\theta = 45\text{–}60^\circ) \sim 0.2$$
$$P(\text{extinction} \mid \text{favourable conditions}) \sim 0.5$$

These factors are independent, so the overall probability is obtained by multiplication. Using mid-range estimates,

$$P(\text{extinction} \mid \text{Chicxulub}) \approx 0.03 \times 0.9 \times 0.2 \times 0.5 = 0.27\%$$

This conditional probability chain shows how rare the precise combination of location, target-rock composition, impact angle, and biological conditions had to be for a single asteroid strike to trigger a global mass extinction

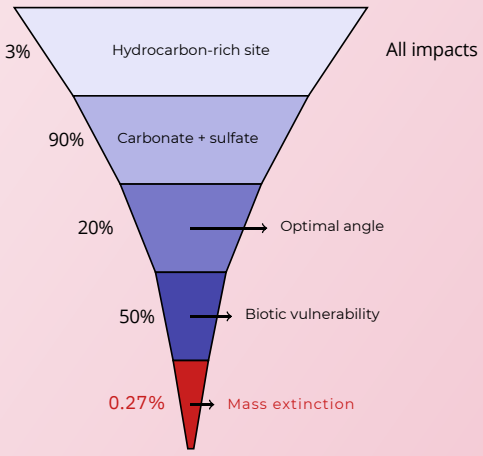


Figure 6: Probability funnel showing how multiple independent factors combine to produce mass extinction. Each condition eliminates the vast majority of scenarios, leaving only 0.27% of large impacts capable of causing global catastrophe.

8 Conclusion

The Chicxulub impact was catastrophic because multiple rare factors: impact angle, velocity, and reactive target rocks, combined to create extreme global environmental effects [19, 2]. It was therefore an improbable event, rather than a typical large-asteroid impact.

Fortunately, the very low probability we calculated earlier (≈ 0.27%) is reassuring, and scientific estimates confirm that the chance of a civilisation-threatening asteroid impact in the foreseeable future remains extremely small [22].

Even more importantly, we now have active detection programs and proven deflection technology. Space agencies such as NASA and ESA track near-Earth objects with telescopes, radars, and upcoming missions like NEO Surveyor [23].

The DART mission in 2022 successfully demonstrated kinetic impact deflection by crashing a spacecraft into the asteroid Dimorphos, shortening its orbit by about 32 minutes and validating this method for potential future threats [24].

The chance of a mass-extinction-level impact is tiny—and humanity is no longer helpless against the rare ones that could appear.

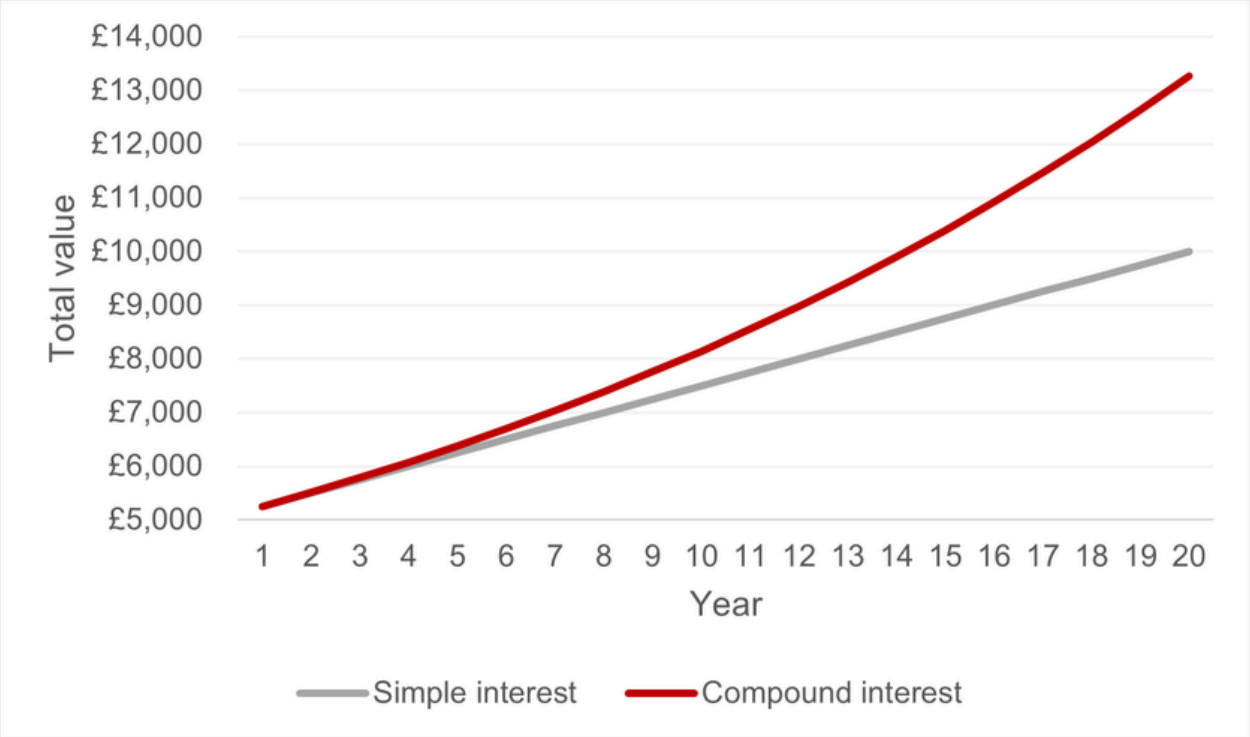


Figure 7: Artist's impression of the Chicxulub asteroid impact. [6]

SMALL ACTS, BIG IMPACT

Emerson Wilson

Introduction
Exploring the maths of how small decisions can lead to big changes in society and some of its positive and negative outcomes.
A simple example of how making small changes can, over time, make a large difference is 'Compound Interest'. Looking at the graph below it shows how the growth of a small interest on an initial investment will accelerate and with time will make a healthy return.



The maths behind this uses this formula

- $A = P(1 + r/n)^{(nt)}$ Where:
- **A** = the future value of the investment/loan, including interest
- **P** = the principal investment amount (the initial deposit or loan amount)
- **r** = the annual interest rate (decimal)
- **n** = the number of times that interest is compounded per year
- **t** = the number of years the money is invested or borrowed for.

It basically means that each year the investor will not only make interest on their initial investment but also on the last and any previous years gains. This means that the gain increases year on year getting larger and larger.

It's easy to see with money how small amounts of change can, when compounded, lead to a large change but there are other areas of society where the same mathematical principle can have

both large positive or negative effects.

As a society many of us rely heavily on social media to share and discover new information and ideas. Imagine some misinformation shared initially by one user to 20 followers. This is then shared with 20 of each of their followers. Just like the compound interest above the speed of the information sharing accelerates, just like a feedback loop. The formula for this is x^n, where x is the number of people shared to each time and n is the number of times shared. Add to this the reinforcement that when a post is shared many times with many 'likes' its perceived as being more reliable and that will in turn mean that more users forward it on. In this instance the information itself will decide if the change to society is negative or positive. A small lie can become a very large one and do a lot of damage but equally a piece of positive information can also spread quickly.

Tim Harford, in his book 'The Logic of Life' looked into an idea introduced by Thomas Schelling who was an economist. This idea was that, in an integrated society, even a very small 'preference' by members of society to live closer to other people 'like them' would always lead to a segregated society. And like the examples above, the more people moved to be closer to neighbours like them, the more prominent these groups of neighbours became and so they became bigger magnets to other people who wanted to live in neighbourhoods of similar people to them. As the segregated neighbourhoods get bigger this then also will encourage people (even with just a mild preference) to then move away to be near more people like them.

In 2003 the British Cycling Team hired David Brailsford and his strategy was "the aggregation of marginal gains,". This basically meant that by making lots of small improvements they will all impact on each other and the overall improvement can be quite significant. Some examples of small improvements they made include optimising sleep for the cyclists, improving hygiene to prevent illness and redesigning aerodynamic gear (helmets etc…). Over the next ten years it became clear that this strategy really did work as the British cycling team rocketed ahead winning more and more medals and setting many new world records.

The positive effect of many small changes can also be seen in motor sport and how they compete by making minute changes to aerodynamics, tiny reductions in the weight of the whole by cutting a few grams of many components, adjustments to tyre pressures, training the driver to make small delays in braking times. These all make a difference on their own but added together they can give a team that extra milliseconds that it needs to win.

Small changes in relation to maths /effort /prejudice can lead to large differences/ gains. Maths helps us explain this and enables us to understand why tiny changes matter. It can help us see that our efforts can lead to great things but also lets us see how easily small thoughts and gestures can have a very large negative effect on society.

THE BIRTHDAY PARADOX:
WHY YOUR BRAIN IS BAD AT MATHS (AND WHY THAT'S OKAY)

Keighley Croot

Imagine you're in a room with 23 strangers. If I told you there's a 50/50 chance that two people in that room share the same birthday, would you believe me?

Most people wouldn't. In fact, when asked how many people it takes to reach those odds, the average person guesses 183—roughly half the number of days in a year. It feels logical, right? But in the world of probability, our "gut feeling" is often dead wrong.

This is the essence of the Birthday Paradox - a striking example of how human intuition often misjudges the likelihood of events. The paradox challenges our understanding of probability and reveals that we're far more likely to encounter shared birthdays than we realize.

The "Me" Trap: Why Our Intuition Gets It Wrong

Why are our intuitions so far off? The answer lies in how we perceive ourselves in a group. When thinking about birthdays, most people focus on themselves.

- **The Individual View:** You walk into a room of 23 people, and your first instinct is to look for someone with the same birthday as you. You have 22 other people to compare yourself to, and the odds of finding a match feel small—around 6%.
- **The Collective View:** But the paradox isn't about you finding a match; it's about whether any pair of people in the room shares a birthday. When we shift from an individual focus to a collective one, the maths gets much more interesting—and much more surprising.

The Power of Connections: How 23 People Lead to 253 Pairs

The key to the paradox lies in the number of possible pairings. While there are only 23 people in the room, the number of ways they can be paired up is much larger than it appears.

- Person 1 can pair with 22 other people.
- Person 2 has already paired with Person 1, but can still pair with 21 new people.
- Person 3 has 20 new people to pair with… and so on.

In total, 23 people can create 253 possible pairs. Instead of focusing on just one match, you're essentially trying to find one shared birthday out of 253 attempts. With so many opportunities to hit a 1-in-365 chance, the 50% probability suddenly starts to make sense.

The Logic Behind the Numbers: Avoiding a Match is Harder Than You Think

To simplify the maths, we'll take an easier route: instead of calculating the odds of a match, we'll calculate the odds that no one shares a birthday and subtract that from 100%. It turns out that it's much simpler to

calculate the likelihood of everyone having a unique birthday than trying to find a pair.
Here's how it works:

- Person 1 walks in. They can have any birthday, so there's a 100% chance their birthday is unique.
- Person 2 walks in. To have a unique birthday, their odds are 364/365 (since one day is already taken).
- Person 3 walks in. They need to have a different birthday from the first two, so their odds are 363/365, and so on.

We keep multiplying these odds together: (365/365) × (364/365) × (363/365) × ... × (343/365)(365/365) × (364/365) × (363/365) × ... × (343/365)

After multiplying all the decreasing fractions for 23 people, we arrive at a probability of approximately **0.49**—meaning there's a 49% chance that everyone has a unique birthday. That leaves a **51%** chance that at least two people share one.

The Real "Magic" of Probability
So, it's not about you finding a birthday twin—it's about 23 people trying to avoid any matches, and that task is harder than it seems. What feels like an unlikely coincidence is actually the result of 253 opportunities for a match to occur, which makes a 50% probability more than reasonable.

Linear Brains in an Exponential World
The Birthday Paradox is a classic illustration of how humans struggle with exponential growth. Our brains are wired to think linearly—add one, then another, then another. But probabilities, especially when they involve pairings or connections, grow exponentially. This "intuition gap" is the same reason people find it hard to grasp:

- How a virus spreads so quickly in a population.
- How compound interest turns a small sum of money into a fortune over decades.
-

In both cases, the growth isn't gradual—it's explosive.

Key Probability Milestones

1. **23 People (50.7% Probability):** This is where it becomes more likely than not that at least two people share a birthday.
2. **57 People (99% Probability):** At this point, a match is almost certain.
3. **70–75 People (99.9% Probability):** In a group this size, the chances of not finding a match are less than 1 in 1,000.
4. **367 People (100% Probability):** With 366 possible birthdays (including February 29), the 367th person guarantees a shared birthday—this is a direct consequence of the Pigeonhole Principle which states that if you have more items (pigeons) than containers (pigeonholes) and put every item into a container, at least one container must hold more than one item.

Why It Matters: Real-World Applications
The Birthday Paradox isn't just a quirky math puzzle. It has profound real-world implications in various fields:

1. **Cybersecurity & Cryptography** The Birthday Paradox underpins a cryptographic threat where attackers use the paradox to find "collisions" in

hash functions. By exploiting the limited number of possibilities, they can generate fraudulent documents that match legitimate ones.

2. `**Forensic Science (DNA Evidence)** : In large DNA databases, even a match that seems incredibly rare (like "1 in a billion") may not be as extraordinary as it seems. The sheer number of DNA profiles in the database increases the likelihood of random matches, leading to potentially faulty conclusions.
3. **Real-World Risk Management**
   - Satellite Collisions: The probability of two specific satellites colliding might be tiny, but with thousands of objects in orbit, the number of potential "pairs" makes collisions an increasing risk.
   - Project Management: As you add more people to a team, the number of communication "connections" grows exponentially. This can overwhelm a team's capacity and create coordination challenges, just as the paradox illustrates.

Conclusion

The Birthday Paradox is a powerful reminder that the world is far more interconnected than we tend to think. By shifting our perspective from the individual to the collective, we realize that what we often perceive as rare or extraordinary is simply the result of hidden statistical patterns. Our brains evolved to help us navigate daily life, but when it comes to understanding probabilities, they often lead us astray. By embracing these surprising mathematical truths, we can gain a deeper, more accurate understanding of the world around us.

